

Hypothesis Tests

ANSWERS

1. At CSLA, the SAT scores of entering students have had a mean of 950 (for the critical reading and mathematics portions). Each year a sample of applications is taken to see whether the examination scores are at the same level as in previous years. A sample of 169 students in this year's class had a sample mean score of 980 and a standard deviation of 150.
 - a. Test the null hypothesis $H_0: \mu=950$ against a two-sided alternative hypothesis. Use a .05 level of significance.
 - b. Calculate the prob value of the test statistic. Interpret.

2. Suppose CSLA tests the idea that computerization of statistics classes increases student performance in the courses. If the computers are found to have a beneficial effect, the school will spend twenty million to computerize all statistics classrooms. The population mean statistics GPA presently is believed to be 2.35. A sample is taken of 100 students who took experimental computerized statistics classes. Their sample GPA in the stat classes was 2.45 and a sample standard deviation $S=1.25$.

Test the idea that computerization increases student GPA by placing the hypothesis that computers have no effect within the null and setting a level of significance of .01.

Should the school computerize the classrooms? Why can you not answer this question with certainty? Explain why such a low level of significance was set. If your conclusion ultimately is in error what type of error is it? Will we ever know if we are in error?

3. The company General Electric claims that a certain brand of its flashlight battery lasts, on average, 300 hours of flashlight use. You suspect that the population of batteries average fewer than 300 hours. You select a random sample of 49 batteries and obtain a sample mean of 290 and a sample standard deviation $S=70$.
 - a. Perform a one-tailed hypothesis test with the company's claim in the null hypothesis. Use a level of significance of .10.
 - b. If your test conclusion is in error, what type of error is it? Type I or II?
 - c. If you did not reject the null hypothesis, what are you concluding about the difference between the sample mean battery lifespan and the company's claim?
 - d. Calculate the prob-value of the test statistic. Interpret.
4. Data published by California on the salaries of state workers indicate the average yearly salary of full-time secretarial staff at state universities is \$47,500. We examine whether the mean salary for secretarial staff at CSULA is different from the state mean. A sample of 64 full time staff members generates a mean of \$49,000 and a standard deviation of \$6,200.
 - a. Describe the population for this problem.
 - b. Perform a two tailed hypothesis test using a level of significance of .10. Interpret.
 - c. Are we concluding that the difference between the test statistic and the hypothesized population mean reflects sampling error? Explain.
 - d. Calculate the prob-value for the test statistic. Interpret.

5. Suppose the firm, *Web Associates*, builds advertising websites for client companies. The firm claims that their websites average 550 *hits*, or visits, per day by potential customers. Suspecting the company is exaggerating, you test this claim by compiling data on the number of hits in a day for a sample of 36 websites developed by *Web Associates*. You calculate a sample mean of 500 *hits* and a sample standard deviation of 120.5.
 - a. Test the company's claim using an alpha of 5%.
 - b. Did you perform a one tailed or two-tailed test? Explain.
 - c. Describe the population you sampled from.
 - d. After performing the test, do we know for sure whether *Web Associates'* claim is factual or not?
 - e. Calculate the prob-value of the test statistic. Interpret.

6. The unemployment rate in California is currently 9.8%. You hypothesize that the rate specifically in Los Angeles County is higher. You base this on the fact that labor market participants in Los Angeles are younger and more likely to be immigrants. You randomly sample 850 labor market participants in Los Angeles and find that 100 are unemployed.
 - a. Describe in detail the population you sampled.
 - b. Set up the test. Why is this test constructed as a one-tailed test?
 - c. Perform the hypothesis test using a 1% level of significance.
 - d. Do we know for sure whether LA County's unemployment rate is higher than 9.8%?
 - e. Explain why we would rather have a larger sample than a smaller one to perform hypothesis tests.

7. Suppose a liberal arts university has long boasted that 65% of its graduating students immediately go on to graduate school. The school decides to perform a hypothesis test to see if it is exaggerating. It samples 300 recent graduates and finds that 165 of them went on to graduate school.
 - a. Describe the population that is sampled.
 - b. Perform a one tailed hypothesis test placing the university's boast within the null hypothesis. Use a level of significance of .05.
 - c. Are we concluding that the difference between the test statistic and population proportion reflect sampling error?
 - d. Should the school continue boasting that sixty-five percent of its graduates go immediately on to graduate school?

1. At CSLA, the SAT scores of entering students have had a mean of 950 (for the critical reading and mathematics portions). Each year a sample of applications is taken to see whether the examination scores are at the same level as in previous years. A sample of 169 students in this year's class had a sample mean score of 980 and a standard deviation of 150.
 - a. Test the null hypothesis $H_0: \mu=950$ against a two-sided alternative hypothesis. Use a .05 level of significance.

The two critical values of the test are 927.38 and 972.62. We reject the null hypothesis because the test statistic is in the rejection region on the right hand side of the distribution.
 - b. Calculate the prob value of the test statistic. Interpret.

The prob value is .0094. There is a .94% probability of obtaining an sample mean value further from the hypothesized population mean than our test statistic was from the mean.

2. Suppose CSLA tests the idea that computerization of statistics classes increases student performance in the courses. If the computers are found to have a beneficial effect, the school will spend twenty million to computerize all statistics classrooms. The population mean statistics GPA presently is believed to be 2.35. A sample is taken of 100 students who took experimental computerized statistics classes. Their sample GPA in the stat classes was 2.45 and a sample standard deviation $S=1.25$.

Test the idea that computerization increases student GPA by placing the hypothesis that computers have no effect within the null and setting a level of significance of .01.

The critical value for this one tailed on the right side is 2.64. The test statistic is 2.45 which leads us to accept the null hypothesis.

Should the school computerize the classrooms? No. Why can you not answer this question with certainty? Because the decision is based on sample data. There is a chance we are making a type II error. Explain why such a low level of significance was set. To minimize the chance of making a type I error. If your conclusion ultimately is in error what type of error is it? Type II. Will we ever know if we are in error? Probably not, given that we will not put computers in the classroom.

3. The company General Electric claims that a certain brand of its flashlight battery lasts, on average, 300 hours of flashlight use. You suspect that the population of batteries average fewer than 300 hours. You select a random sample of 49 batteries and obtain a sample mean of 290 and a sample standard deviation $S=70$.
- Perform a one-tailed hypothesis test with the company's claim in the null hypothesis. Use a level of significance of .10.
The left-hand side critical value of the test are 287.2. The test statistic of 290 is within the acceptance region so we accept the null hypothesis.
 - If your test conclusion is in error, what type of error is it? Type I or II?
Type II
 - If you did not reject the null hypothesis, what are you concluding about the difference between the sample mean battery lifespan and the company's claim?
We are concluding that it is sampling error, although we might be wrong.
 - Calculate the prob-value of the test statistic. Interpret.
There is a 15.87% probability of obtaining an X-bar further from the hypothesized mean than our test statistic was from the mean. The prob-value is greater than the alpha set for the test, which explains why we accepted the null hypothesis.
4. Data published by California on the salaries of state workers indicate the average yearly salary of full-time secretarial staff at state universities is \$47,500. We examine whether the mean salary for secretarial staff at CSULA is different from the state mean. A sample of 64 full time staff members generates a mean of \$49,000 and a standard deviation of \$6,200.
- Describe the population for this problem.
All full time secretarial staff at CSULA.
 - Perform a two tailed hypothesis test using a level of significance of .10. Interpret.
The critical values of the test are \$46,225.13 and \$48,774.88. The test statistic falls into the rejection region, so we reject the null hypothesis.
 - Are we concluding that the difference between the test statistic and the hypothesized population mean reflects sampling error? Explain.
We are concluding that the difference between the test statistic and hypothesized does not reflect sampling error, although we might be wrong.
 - Calculate the prob-value for the test statistic. Interpret.
There is a 5.24% probability of obtaining an X-bar further from the hypothesized mean than our test statistic was from the mean. The prob-value is less than the alpha set for the test, which explains why we rejected the null hypothesis.

5. Suppose the firm, *Web Associates*, builds advertising websites for client companies. The firm claims that their websites average 550 *hits*, or visits, per day by potential customers. Suspecting the company is exaggerating, you test this claim by compiling data on the number of hits in a day for a sample of 36 websites developed by *Web Associates*. You calculate a sample mean of 500 *hits* and a sample standard deviation of 120.5.
- Test the company's claim using an alpha of 5%.
The hypothesis test is $H_0: \mu = 550$ $H_1: \mu < 550$
The critical value of the test is 516.96. The test statistic is in the rejection region, therefore reject H_0 .
 - Did you perform a one tailed or two-tailed test? Explain.
We performed a one tailed test on the left hand side.
 - Describe the population you sampled from.
The population consists of all the websites built by *Web Associates* for clients.
 - After performing the test, do we know for sure whether *Web Associates*' claim is factual or not?
No. Our conclusion is based on sample data.
 - Calculate the prob-value of the test statistic. Interpret.
 $P(\bar{x} < 500) = P(Z < -2.48) = .0066$ There is a .66% chance of obtaining a sample mean further from the hypothesized mean than our test statistic was from the population mean.
6. The unemployment rate in California is currently 9.8%. You hypothesize that the rate specifically in Los Angeles County is higher. You base this on the fact that labor market participants in Los Angeles are younger and more likely to be immigrants. You randomly sample 850 labor market participants in Los Angeles and find that 100 are unemployed.
- Describe in detail the population you sampled.
The labor force in LA County.
 - Set up the test. Why is this test constructed as a one-tailed test?
 $H_0: P = .098$ $H_1: P > .098$ It is constructed as a one tailed test because the researcher suspects the unemployment rate in LA county is higher than 9.8%.
 - Perform the hypothesis test using a 1% level of significance.
The critical value of the test is 0.122. The test statistic value is 0.118 therefore do not reject H_0 .
 - Do we know for sure whether LA County's unemployment rate is higher than 9.8%?
No. We based our conclusion on sample data.
 - Explain why we would rather have a larger sample than a smaller one to perform hypothesis tests.
Larger samples lead to smaller standard errors for our parameter estimates. A larger sample implies we are less likely to be far off in our estimate of the population proportion.

7. Suppose a liberal arts university has long boasted that 65% of its graduating students immediately go on to graduate school. The school decides to perform a hypothesis test to see if it is exaggerating. It samples 300 recent graduates and finds that 165 of them went on to graduate school.
- Describe the population that is sampled.
All graduates (BA degree) over a given time period from the liberal arts school.
 - Perform a one tailed hypothesis test placing the university's boast within the null hypothesis. Use a level of significance of .05.
 $H_0: P=.65$ $H_1: P<.65$
The critical value is 0.605; the test statistic is .55, therefore reject H_0 .
 - Are we concluding that the difference between the test statistic and population proportion reflect sampling error?
No, we are saying that the difference between the test statistic and hypothesized population parameter reflects the null hypothesis being incorrect.
 - Should the school continue boasting that sixty-five percent of its graduates go immediately on to graduate school?
No, we are not confident that the boast is factual.