The firm’s Long Run Costs

Due to scarcity, firm must incur cost to divert resources from alternative uses

Define:

w – hourly cost of labor

reflects firm compensating owners of labor for forgoing next best use of time

r – hourly rental rate of capital

Consider decision to employ K capital and L labor to produce Q bicycles per time period

Opportunity cost of decision:

Explicit – cost in form of direct dollar payments to input owners; money is transferred between buyer and seller

Implicit – cost reflecting foregone opportunities, but not involving actual transfers of dollar payments

[Examples in opportunity cost](http://milesfinney.net/410/handout/oppcost.htm)

In long run firm will choose input mix that minimizes opportunity cost

Firm’s total cost identity: TC=wL+rK

Isocost line

identifies input combinations firm could purchase at a given total cost

K

L

Isocost Line

$$\frac{TC}{r}$$

$$\frac{TC}{w}$$

total cost constant at every point along isocost line

slope of isocost is $\frac{w}{r}$

suppose w=$20/hour and r=$5/hour

$\frac{w}{r}$ = $\frac{20}{5}$ =4

Interpret: for each unit hour of labor utilized, the firm would have to forego 4 hours of capital to maintain constant total cost

Firm will choose input mix that minimizes total cost

Isocost line is not a budget constraint for firm

The firm has control over costs it incurs

Suppose firm wants to produce Q0 bicycles

K

L

$$\frac{TC}{r}$$

$$\frac{TC}{w}$$

a

Ka

La

Q0

Input mix a is cost minimizing

Every other input mix along Q0 costs more – corresponds to higher isocost

Only one condition at point a:

Slope of isoquant equals slope of isocost

MRTSL,K = $\frac{w}{r}$

**Derivation of the firm's cost function**

Suppose the firm produces Q0 at the cost minimizing input mix "a".

(Capital) K

Ka

a

Q0

La

L (Labor)

Assume the firm's production function: *Q=F(K,L)=K1/2L1/2*

The slope of the isoquant reveals the marginal rate the firm is able to substitute between inputs holding output constant. The marginal rate of technical substitution (MRTS) is calculated as the ratio of marginal products of capital (K) and labor (L):

 $MRTS=\frac{MP\_{L}}{MP\_{K}}$ where $MP\_{L}=\frac{∂F\left(K,L\right)}{∂L}=\frac{1}{2}\left(\frac{K}{L}\right)^{^{1}/\_{2}}$

$MP\_{K}=\frac{∂F\left(K,L\right)}{∂K}=\frac{1}{2}\left(\frac{L}{K}\right)^{^{1}/\_{2}}$

MRTS = $\frac{K}{L}$

We know that at the cost minimizing input mix, the marginal rate of technical substitution (MRTS) equals the slope of the isocost line.

at point “a”:

(1) MRTS = $\frac{w}{r}$ or $\frac{K}{L}= \frac{w}{r}$

Solve for K to find the relationship between capital and labor that must hold at every cost minimizing output level.

(2) K = $\frac{w}{r}L$

Substitute this equation for K into the production function.

(3) Q = $\left(\frac{w}{r}L\right)^{^{1}/\_{2}}L^{^{1}/\_{2}}$

 = $\left(\frac{w}{r}\right)^{^{1}/\_{2}}L$

If you take equation 3 and solve for L you get:

(4) L = Q$\left(\frac{r}{w}\right)^{^{1}/\_{2}}$

Use equation 4 to substitute for L in equation 2:

(5) K = $\frac{w}{r}Q\left(\frac{r}{w}\right)^{^{1}/\_{2}}$

 = Q$\left(\frac{w}{r}\right)^{^{1}/\_{2}}$

Equations 4 and 5 are the demand for capital and labor as a function of input prices and output level.

The firm's total cost can be represented as the sum of total expenditure on its respective inputs.

(6) *TC = rK + wL*

To derive the equation for the firm's long run total cost, substitute the functions for L and K in 4 and 5 into equation 6:

(7) TC = rQ$\left(\frac{w}{r}\right)^{^{1}/\_{2}}+$ wQ$\left(\frac{r}{w}\right)^{^{1}/\_{2}}$

 = Q$\left(rw\right)^{^{1}/\_{2}}$ + Q$\left(rw\right)^{^{1}/\_{2}}$

 = 2 Q$\left(rw\right)^{^{1}/\_{2}}$

1. Take the derivative of the total cost function with respect to w. Does the resulting equation look familiar? Do the same with respect to r.

2. Take the derivative of the total cost function with respect to Q. Compare this to the average cost function. What does the result say about returns to scale?

K

L

$$\frac{TC}{r}$$

$$\frac{TC}{w}$$

a

Ka

La

Q0=50

.

b

Find cost minimizing mix of labor and capital to produce Q0=50 bikes per time period (input mix a). w=$20/hour and r=$5/hour

1. Confirm that at the cost minimizing mix:
	1. F(K,L) = Q0
	2. MRTSL,K = $\frac{w}{r}$
2. Calculate total cost at the cost minimizing mix.

Suppose input mix b above corresponds to K=125

1. Calculate labor use at point b.
2. Calculate MRTS at point b.
3. Confirm MRTSL,K >$\frac{w}{r}$ at point b. Interpret
4. Calculate total cost at point b. Use marginal relationship to explain why TC at point b is higher than at point a.

Total cost function calculates minimum cost of achieving a given output level

Condition demand for inputs may be derived from TC function (Shepard’s Lemma)

$\frac{∂TC}{∂w}$ - conditional demand function for labor

$\frac{∂TC}{∂r}$ - conditional demand function for capital

For bicycle manufacturer:

TC = 2 Q$\left(rw\right)^{^{1}/\_{2}}$

L = Q$\left(\frac{r}{w}\right)^{^{1}/\_{2}}$

K = Q$\left(\frac{w}{r}\right)^{^{1}/\_{2}}$

For profit maximizing firm, interpret and confirm conditions:

$\frac{∂L}{∂w}$ < 0 $\frac{∂K}{∂r}$ < 0

Effect of change in input price:

K

L

e'

Ke’

Le’

Q0

Ke

Le

e

TC1

TC2

Firm produces Q0 using input mix Ke, Le

Suppose price of labor increases (w↑)

1. Total cost of producing Q0 would increase.

$\frac{∂TC}{∂w}$ >0

1. Amount of capital used to produce Q0 will increase

$\frac{∂K}{∂w}$ >0

1. Labor use decreased

$\frac{∂L}{∂w}$ <0

Firm would move from point e to e’

Isocost TC1 to TC2 where TC2>TC1

Firm responds to wage increase by substituting labor for capital

Input mix e’ is now the least cost method to produce Q0

[Example of Nepal](http://milesfinney.net/410/handout/oppcost.htm)

Nepal low w; high r

U.S. high w; low r