**Derivation of Demand Function for the Utility Maximizing Consumer**

Let’s take the consumer illustrated in class who is maximizing utility by consuming food and shelter.

Food

Shelter

M/PF

M/PS

U1

Sa

Fa

a

Two conditions hold at basket “a” where the consumer maximizes utility. First, the consumer’s income equals expenditures. Second, the consumer’s marginal rate of substitution equals the slope of the budget line.

The first condition is represented by equation (1).

1. M = PFF+PSS

where M is consumer income; PF and PS are the price per unit of food (F) and shelter (S).

Rearranging terms yields the simple formula for a budget line.

1. $F=\frac{M}{P\_{F}}-\frac{P\_{S}}{P\_{F}}S$ where $\frac{P\_{s}}{P\_{F}}$ is the slope of the budget line.

Suppose the consumer’s preferences are represented by the function,

1. U(F,S)=F·S

The consumer’s marginal rate of substitution (MRSS,F) is the slope of the indifference curve and is measured by the ratio of the goods’ marginal utilities.

$MRS\_{S,F}=\frac{-∆F}{∆S}=\frac{MU\_{S}}{MU\_{F}}$ where $MU\_{S}=^{∂U(∙)}/\_{∂S}=F$

 $MU\_{F}=^{∂U(∙)}/\_{∂F}=S$

$$MRS\_{S,F}=\frac{F}{S}$$

Therefore, the second condition for utility maximization - the marginal rate of substitution equaling the slope of the budget line - is shown for this consumer in equation (4).

1. $\frac{F}{S}=\frac{P\_{S}}{P\_{F}}$

Equation (4) indicates the ratio of food to shelter units consumed will always be perfectly inverse to the price ratio at the utility maximizing basket. For example, if the per unit price of food is twice that of shelter, this consumer in equilibrium will consume half as many units of food as shelter. This consumption pattern is specific to this particular consumer’s preferences – another utility function may indicate different behavior.

To derive the demand function for food and shelter, we must incorporate the consumer’s income constraint. Go back to the budget constraint equation (2) and, using equation (4), substitute $\frac{F}{S}$ for the budget line slope $\frac{P\_{S}}{P\_{F}}$, as shown in (5).

1. F = $\frac{M}{P\_{F}}-\left(\frac{F}{S}\right)S$

Equation (5) reduces to the consumer’s demand function for food.

1. F = $\frac{M}{2P\_{F}}$

To derive the demand for shelter, use equation (6) as a substitute for *F* in the budget constraint equation (1).

1. M = $P\_{F}\left(\frac{M}{2P\_{F}}\right)+P\_{S}S$

Reduce and solve for *S* to get the demand equation for shelter.

1. S = $\frac{M}{2P\_{S}}$

A. Demand equations represent individual consumer’s utility maximizing behavior.

B. The equations solve for the utility maximizing consumption of F and S given prices and income.

C. The pairs of F and S solved for are equilibrium points on an indifference curve/budget line diagram (such as point *a* on the diagram above).

1. Confirm that at the consumer’s equilibrium

A. Income equals expenditures.

B. The marginal condition MRSS,F = $\frac{P\_{S}}{P\_{F}}$ holds.

2. The downward sloping indifference curve shown above indicates the consumer is willing to substitute between food and shelter in maximizing utility. However, looking at the demand equations (6) and (8), the cross price effects are nonexistent: $\frac{∆F}{∆P\_{S}}$ and $\frac{∆S}{∆P\_{F}}$ equal zero. Are these goods substitutes or aren’t they? Use the idea of substitution and income effects of price changes to explain the apparent contradiction.

Why is basket K(2,9) below not considered an equilibrium?

Food

Shelter

M/PF

M/PS

U1

10

5

a

K

9

2

U2

Income is exhausted at point K but the marginal condition (MRSS,F = $\frac{P\_{S}}{P\_{F}}$)

does not hold.

* at K MRSS,F = 4.5 > $\frac{P\_{S}}{P\_{F}}$ (=1/2)
* consumer is willing to forego approximately 4.5 pounds of food to acquire the next square yard of shelter, holding utility constant
* consumer must forego ½ pound food in market to acquire next square yard of shelter

Consumer willingness to forego food for shelter is greater than opportunity cost to consumer of doing so

Therefore consumer will move away from K substituting food for shelter